A **prefix sum array** is a data structure that allows for efficient computation of the sum of elements in a subarray. The prefix sum array is constructed from an original array, where each element at index **i** in the prefix sum array represents the sum of all elements from the start of the original array up to index **i**.

### **Construction of a Prefix Sum Array**

Given an array **A** of size **n**, the prefix sum array **P** is defined as follows:

* ( P[0] = A[0] )
* ( P[i] = P[i-1] + A[i] ) for ( 1 \leq i < n )

This means that each element in the prefix sum array is the cumulative sum of the elements in the original array up to that index.

### **Example**

Consider the array **A = [1, 2, 3, 4, 5]**. The prefix sum array **P** would be:

* ( P[0] = 1 )
* ( P[1] = 1 + 2 = 3 )
* ( P[2] = 1 + 2 + 3 = 6 )
* ( P[3] = 1 + 2 + 3 + 4 = 10 )
* ( P[4] = 1 + 2 + 3 + 4 + 5 = 15 )

Thus, the prefix sum array **P** is **[1, 3, 6, 10, 15]**.

### **Applications of Prefix Sum Arrays**

1. **Range Sum Queries**: The primary application of a prefix sum array is to quickly compute the sum of elements in a subarray. For a subarray from index **l** to **r**, the sum can be computed in constant time ( O(1) ) using: [ \text{sum}(A[l \ldots r]) = P[r] - P[l-1] ] (with a special case for when ( l = 0 ), where the sum is simply ( P[r] )).
2. **Frequency Counting**: Prefix sums can be used to count occurrences of elements in a range, especially in problems involving cumulative frequencies.
3. **Dynamic Programming**: In some dynamic programming problems, prefix sums can help optimize the computation of certain states by reducing the number of operations needed to calculate sums.
4. **2D Prefix Sums**: The concept can be extended to two dimensions, where a 2D prefix sum array allows for efficient computation of sums in a submatrix.
5. **Algorithm Optimization**: Prefix sums can be used to optimize algorithms that require repeated summation over the same array, reducing the time complexity from ( O(n) ) for each query to ( O(1) ) after an initial ( O(n) ) preprocessing step.
6. **Data Analysis**: In data analysis and statistics, prefix sums can be used to compute cumulative distributions and other statistical measures efficiently.

Overall, prefix sum arrays are a powerful tool in algorithm design, particularly for problems involving range queries and cumulative calculations